

2025年度実施

慶應義塾大学大学院入試問題

経済学研究科（修士課程）

2025年7月5日 実施

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| 受験番号 | Examination number | 氏名 | Name |
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注意事項 (Please note:)

1. This set of questions contains 10 pages (including the cover page).
2. There are seven questions from which you should choose two to answer. Each question should be answered on a separate answer sheet. Please write the number of the question you are answering on each answer sheet.
3. If you answer two or more questions on one answer sheet, only the first answer will be treated as a valid answer. Everything after the first answer will not be marked.
4. Answer in English.
5. Although the question sheets will not be collected after the examination, please write your name and examination number (受験番号, jyuken-bango) on the cover page.

**Question 1:** Choose one of the following two problems, A or B, and answer it. If you answer both, all your answers to Question 1 will be considered invalid.

**A:** Suppose there are  $J \in \mathbb{N}$  different products, and each consumer either purchases **at most one unit of a single product** or chooses not to purchase anything.

Let  $x_j$  denote the attributes of product  $j$  that are observable to both the researcher and the consumers, and let  $\xi_j$  denote product  $j$ 's attributes that are observable to the consumers but unobservable to the researcher.

Assume there exists a *continuum of consumers with measure one*, and let  $\pi_j$  denote the share of consumers who choose product  $j \in J$ . We assume that this share is given by:

$$\pi_j = \frac{\exp(x_j\beta - \alpha p_j + \xi_j)}{1 + \sum_{j' \in J} \exp(x_{j'}\beta - \alpha p_{j'} + \xi_{j'})}$$

Note that  $\beta$ ,  $\alpha$ , and  $\xi_j$  are not directly observable to the researcher, whereas  $\pi_j$  is observable through consumer choices.

**A-1:** When the observable attribute  $x_j$  of product  $j$  marginally increases, determine how much the consumer's *willingness to pay* increases. (You may use any variables as needed, regardless of whether they are directly observable.)

**A-2:** Express the share of consumers who choose not to purchase any product,  $\pi_0$ , using only observable variables.

**A-3:** Express the share of consumers who choose not to purchase any product,  $\pi_0$ , using the terms  $x_j\beta - \alpha p_j + \xi_j$ , for  $j \in J$ .

**A-4:** For product  $j$ , express the *own-price elasticity*  $\epsilon_{jj}$  and the *cross-price elasticity*  $\epsilon_{jk}$  using only observable variables and/or the parameter  $\alpha$ .

**A-5:** For any product  $j \in J$ , express  $\delta_j \equiv x_j\beta - \alpha p_j + \xi_j$  using only observable variables.

**A-6:** Consider the case where  $J$  consists of iPhone ( $I$ ), Galaxy (Android) ( $G$ ), Google Pixel (Android) ( $P$ ). When the price of Google Pixel ( $P$ ) increases, explain how the demands for the other two products (iPhone and Galaxy) change, using your answer to **A-3**. Also, discuss whether the predicted changes are consistent with economic intuition.

B. Consider a model of optimal growth.

$$\begin{aligned} \max_{C_t, K_{t+1}} \quad & \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} U(C_t) \\ \text{s.t.} \quad & K_{t+1} = w_t + (1+r_t-\delta)K_t - C_t \end{aligned}$$

Time is discrete, and the utility of the representative household at time  $t$  is given by  $U(C_t)$ , where  $U'(C_t) > 0$  and  $U''(C_t) < 0$ .  $C_t$  represents consumption of the representative household at time  $t$ , and  $K_t$  represents the capital stock at time  $t$ .  $K_0$  is given as an initial condition. The wage rate is denoted by  $w_t$ , and the capital rental rate is denoted by  $r_t$ . The household supplies labor inelastically, and this amount is normalized to 1. The capital stock depreciates at a constant rate of  $\delta$ . The subjective discount rate is  $\rho$  and remains constant over time.

1. Solve the above problem using Lagrange multiplier  $\lambda_t$ , and derive the first-order conditions.
2. Let the production function of the representative firm be  $Y_t = F(K_t, L_t)$ , where  $K_t$  is the total capital,  $L_t$  is the total labor, and the production function exhibits constant returns to scale. Set up the firm's profit maximization problem and derive the first-order conditions. Then provide an interpretation for the wage rate  $w_t$  and the capital rental rate  $r_t$ .
3. Eliminate the Lagrange multiplier from the first-order conditions of the representative household to derive the Euler equation for consumption.
4. Assume the utility function is  $U(C_t) = \ln(C_t)$  and the production function is  $Y_t = K_t^\alpha L_t^{1-\alpha}$  (where  $L_t = 1$ ). Explain the steady state where  $C_t$  and  $K_t$  are constant, using equations and diagrams.

Question 2.

Answer the following questions on capitalism. Base your answer on the methodology of Marxian economics.

(1) Briefly explain the following concepts.

- (a) Value of labor-power
- (b) Surplus-value
- (c) Organic composition of capital
- (d) Primitive accumulation of capital

(2) Explain industrial capital, commercial capital, and interest-bearing capital.



Question 3.

(a) Consider the following definitions:

- Let  $S$  be a set of real numbers. The set  $S$  is **open** if for any  $x \in S$ , there is  $\delta > 0$  such that

$$|x - y| < \delta \Rightarrow y \in S.$$

- Let  $f$  be a function from the set of real numbers to itself. The function  $f$  is **continuous** if for any set  $S$  of real numbers,

$$S \text{ is open} \Rightarrow f^{-1}(S) \text{ is open,}$$

$$\text{where } f^{-1}(S) := \{x \mid f(x) \in S\}.$$

Using the definitions above, answer the following questions:

1. Prove that the intersection  $S_1 \cap S_2$  of two open sets  $S_1$  and  $S_2$  of real numbers is open.
2. Provide an example of a countable sequence  $\{S_n\}_{n=1}^{\infty}$  of open sets of real numbers such that its ~~union~~<sup>intersection</sup>  $\bigcap_{n=1}^{\infty} S_n$  is not open.
3. Prove that the following function  $g$  is not continuous:

$$g(x) = \begin{cases} x & \text{if } x \leq 1, \\ x + 1 & \text{if } x > 1. \end{cases}$$

(b) Use Cramer's rule to solve the following system of equations:

$$\begin{aligned} 2x - 3y &= 2, \\ 4x - 6y + z &= 7, \\ x + 10y &= 1. \end{aligned}$$

Question 4.

Let  $W$  be a random variable satisfying  $E(W^2) < \infty$ . Then the Chebyshev's inequality states that for any real number  $t > 0$

$$P\{|W - E(W)| \geq t\} \leq \frac{V(W)}{t^2}$$

where  $E(\cdot)$  and  $V(\cdot)$  denote the expectation and variance, respectively. Answer the questions (1) and (2) using the Chebyshev's inequality.

(1) Suppose the random variables  $Z_1, Z_2, \dots$  are identically and independently distributed (i.i.d.) and satisfy  $E(Z_i^2) < \infty$ . Then prove that

$$\frac{1}{n} \sum_{i=1}^n Z_i \xrightarrow{P} E(Z_1), \quad n \rightarrow \infty$$

where  $\xrightarrow{P}$  means the convergence in probability.

(2) Suppose that the two dimensional random vectors  $(Z_1, \tilde{Z}_1), (Z_2, \tilde{Z}_2), \dots$  are i.i.d. and satisfy  $E(Z_i^4) < \infty$  and  $E(\tilde{Z}_i^4) < \infty$ . Then prove that

$$\frac{1}{n} \sum_{i=1}^n (Z_i + \tilde{Z}_i)^2 \xrightarrow{P} E\{(Z_1 + \tilde{Z}_1)^2\}, \quad n \rightarrow \infty.$$

Next, suppose that  $n$  observations  $\{(Y_i, X_i, A_i)\}_{i=1}^n$  follow the linear regression model  $Y_i = X_i\alpha + A_iX_i\beta + \varepsilon_i$  where  $X_i$  and  $\varepsilon_i$  are real valued random variables and  $A_i$  is a binary (0 or 1) random variable satisfying that

$$E(X_i) = E(\varepsilon_i) = 0, \quad V(X_i) = V(\varepsilon_i) = 1, \quad P(A_i = 1) = P(A_i = 0) = \frac{1}{2}.$$

Let  $T_i = 2(2A_i - 1)$  and assume that  $X_i, A_i, \varepsilon_i$  are mutually independent. Then answer the following questions (3)–(6).

(3) Show  $E(T_i Y_i | X_i) = X_i \beta$ .

(4) Let  $\hat{\beta}$  be the estimator minimizing the squared sum  $\sum_{i=1}^n (T_i Y_i - X_i \beta)^2$  with respect to  $\beta$ . Then show  $\hat{\beta} \xrightarrow{P} \beta, n \rightarrow \infty$ .

(5) Derive the asymptotic variance of  $\sqrt{n}(\hat{\beta} - \beta)$  when  $n \rightarrow \infty$  and  $X_i$  follows  $N(0, 1)$ .

(6) Let  $(\tilde{\alpha}, \tilde{\beta})$  be the estimators minimizing the sum of squared residuals  $\sum_{i=1}^n (Y_i - X_i\alpha - A_iX_i\beta)^2$  with respect to  $(\alpha, \beta)$ . Derive the asymptotic variance of  $\sqrt{n}(\tilde{\beta} - \beta)$  and then compare it with (5) when  $n \rightarrow \infty$  and  $X_i$  follows  $N(0, 1)$ .

Question 5.

Answer only one of A or B. If you answer both, all answers to Question 5 will be invalid.

A

Answer all the following questions (1)-(5).

- (1) In the standard Mundell-Fleming model, under which exchange rate regime does fiscal expansion have a greater impact on GDP, a fixed exchange rate regime or a floating exchange rate regime? Explain your answer with reasons.
- (2) Explain the Balassa-Samuelson effect. Using the case of two countries (advanced and developing) and two sectors (tradable and non-tradable), explain the mechanism through which the Balassa-Samuelson effect arises.
- (3) Suppose the interest rate on the Japanese yen is 1%, the interest rate on the US dollar is 3%, and the spot exchange rate is 1USD=122JPY. Assuming that covered interest parity holds, compute the 1-year forward exchange rate.
- (4) Suppose Japan's foreign assets amount to 10 billion yen and foreign liabilities amount to 5 billion yen. Further, assume that 50% of the foreign assets and 80% of the foreign liabilities are denominated in US dollars. If the Japanese yen depreciates against the US dollar by 20%, by how much does Japan's net foreign asset position change in Japanese yen terms?
- (5) Over the past one year, suppose the inflation rate in Japan was 1% and the inflation rate in the US was 5%. Assuming that relative purchasing power parity holds, by how much has the exchange rate changed over the past year?

B

Suppose that the home country and the foreign country trade good  $h$  and good  $f$  with each other. Assume that the two countries are symmetric: each country has the same demand function for good  $i \in \{h, f\}$ , given by  $q_i^D = 11 - p_i$ . Due to technological constraints, good  $h$  is produced only in the home country and its supply is fixed at 6 units, while good  $f$  is produced only in the foreign country and its supply is fixed at 6 units.

- (1) For good  $f$ , derive the import demand function  $MD_f$  in the home country and the export supply function  $XS_f^*$  in the foreign country.
- (2) Under free trade, calculate the home country's gains from trade as the sum of the consumer surplus from imports of good  $f$  and the producer surplus from exports of good  $h$ .
- (3) Now suppose that the home country imposes a specific tariff  $t$  on imports of good  $f$ , and the foreign country imposes a specific tariff  $t^*$  on imports of good  $h$ . For simplicity, assume that both countries can choose only one of two tariff rates: 0 (free trade) or 2 (optimal tariff). That is, the home country's strategy is  $t \in \{0, 2\}$ , and the foreign country's strategy is  $t^* \in \{0, 2\}$ . Consider this as a tariff game. Complete the following payoff matrix, where for each combination of strategies  $(t, t^*)$  the payoff of each country is given by its gains from trade (i.e., the sum of the surplus from imports and the surplus from exports). Note that, when the tariff rate is 2, tariff revenue arises and must be included in the calculation of gains from trade.

|         | $t^* = 0$ | $t^* = 2$ |
|---------|-----------|-----------|
| $t = 0$ |           |           |
| $t = 2$ |           |           |

- (4) Find the Nash equilibrium of the tariff game described in (3). Based on this equilibrium, discuss whether the outcome may reflect a "tariff war" in which trade volumes shrink and both countries are made worse off, compared to the case of free trade.

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Question 6.

Choose a specific region or country and discuss how labor mobility has affected its economy, using concrete historical evidence to support your analysis.

Question 7. Answer the following two questions.

(1) Choose either a) or b) and outline the history of economics relating to it (chiefly focus on the period from the 18<sup>th</sup> century to the first half of the 20<sup>th</sup> century).

\*If you choose both a) and b), your answer will be invalid.

a) Value Theory

b) Quantity Theory of Money

(2) Choose one from c), d) or e) and explain the view one figure in the history of economics held on it.

\*If you choose more than one, your answer will be invalid.

c) Poverty

d) Education

e) Technological Progress