2025　Keio University
Graduate School of Economics
General Admissions

Master’s Program: Application Period I
Subjects: Economics

Sample Answers:
Question 3, 4, 5-A, 5-B

Purpose of the Questions:
Question 1-A, 1-B, 2, 6, 7

Question 1-A

This set of questions is designed to assess whether candidates can accurately grasp and formally express fundamental concepts and definitions, without being misled by the superficial form of equations.

A-1: The aim is to verify whether candidates can formally define what Willingness-To-Pay (WTP) represents conceptually. Rather than simply understanding it in words, candidates are expected to derive its formal definition as "the permissible increase in price that keeps the purchase probability constant."

A-2 and A-3: These questions test candidates' understanding of the structure of the given model. In particular, they examine whether candidates can appropriately rewrite the model under the constraint of "using only observable values."

A-4: This question assesses whether candidates have a correct understanding of the basic economic concept of price elasticity of demand. In addition, it tests mathematical skills, including handling exponential and logarithmic expressions accurately.

A-5: This question examines whether candidates can think critically about how to extract information on unobservable model parameters (structural parameters) from observable values.

A-6: This question evaluates whether candidates can correctly interpret the results derived from the model and explain whether these results are consistent with economic intuition. It aims to test not only mathematical correctness but also the ability to discuss intuitive interpretations and economic implications.

Summary

Through these questions, the overall objective is to evaluate candidates' ability to understand the economic concepts and logical structure underlying the model, and to express them precisely—not merely to perform mechanical manipulations of equations.

Question1-B

This problem is on the standard optimal growth model, asking to set up the basic optimization problem, derive the optimality conditions, and provide an appropriate economic interpretation.

Question 2.

(1) The explanation of each concept should relate to the following points.
(a) Value of labor-power
• Labor-power is the aggregate of the physical and mental capacities existing in a human being.
• Like any other commodity, the value of labor-power is determined by the socially necessary labor time required for its production.
• Labor-power is produced through the reproduction or maintenance of the worker. Since workers require a definite quantity of means of subsistence for their maintenance, the value of labor-power is determined by the value of those means.
(b) Surplus value
• Surplus value is the surplus portion of the value created in the use (consumption) of the commodity labor-power—that is, by labor—that exceeds the value of that labor-power.
(c) Organic composition of capital
• The organic composition of capital is the value composition that reflects the technical composition of capital.
• The technical composition of capital refers to the ratio between the total mass of means of production and the total amount of labor employed.
• The value composition of capital refers to the ratio, in value terms, of constant capital to variable capital.
(d) Primitive accumulation of capital
• Primitive accumulation of capital is the historical process—triggered by state intervention and violence—whereby the relation between capital and wage labor is established through the expropriation and concentration of the means of production.

(2) The explanation should relate to the following points.
• Capital moves by repeatedly traversing the circuit G (money)–W (commodity) … P (production process) … W′–G′, with $G' = G + g$, where g is the increment (surplus value / profit).
• Among these forms, the capital that produces surplus value by investing in means of production and labor-power is called industrial capital.
• Because industrial capital cannot commence its next movement if the realization of W′–G′ is delayed, it requires capital—a form of capital that specializes in selling the commodities it has produced; this is called commercial capital.
• Through the mediation of commercial capital, industrial capital initiates a new circuit and, by increasing the rate of turnover, secures greater profit; accordingly, a portion of profit is distributed to commercial capital.
• Industrial and commercial capital require funds at each phase of their movement. Banks concentrate idle money scattered throughout society and lend to industrial and commercial capital, thereby appropriating a portion of profit as interest—ultimately derived from the surplus value created in production. Such capital is called interest-bearing capital.

Question 3.

(a) 1. Let $x \in S_1 \cap S_2$. Since S_1 is open, there exists $\delta_1 > 0$ such that $|x - y| < \delta_1 \Rightarrow y \in S_1$. Since S_2 is open, there exists $\delta_2 > 0$ such that $|x - y| < \delta_2 \Rightarrow y \in S_2$. Let $\delta_3 = \min\{\delta_1, \delta_2\}$. Then, we have $\delta_3 > 0$, and $|x - y| < \delta_3 \Rightarrow y \in S_1 \cap S_2$.

2. For every natural number n, let $S_n := (0, 1 + \frac{1}{n})$. Let $x \in S_n$. If we choose $\delta := \frac{\min\{x, 1+\frac{1}{n}-x\}}{2}$, then we have $\delta > 0$ and $|x - y| < \delta \Rightarrow y \in S_n$. Thus, S_n is open. Note that, for every n, $1 \in S_n$. Thus, $1 \in \cap_{n=1}^{\infty} S_n$. On the other hand, for every real number $\delta > 0$, there exists a natural number n such that $0 < \frac{1}{n} < \frac{\delta}{2}$, and hence, $1 + \frac{\delta}{2} \notin S_n$. Thus, for every $\delta > 0$, we have $|1 - (1 + \frac{\delta}{2})| = \frac{\delta}{2} < \delta$ and $1 + \frac{\delta}{2} \notin \cap_{n=1}^{\infty} S_n$. Therefore, $\cap_{n=1}^{\infty} S_n$ is not open.

3. Let $S := (0, \frac{3}{2})$. Then $g^{-1}(S) = (0, 1]$. Note that, for every $y \in S$, if we choose $\delta := \frac{\min\{y, \frac{3}{2}-y\}}{2}$, then we have $\delta > 0$ and $|y - y'| < y \Rightarrow y' \in S$. Thus, S is open. However, although $1 \in (0, 1]$, for every $\delta' > 0$, we have $|1 - (1 + \frac{\delta}{2})| = \frac{\delta}{2} < \delta$ and $1 + \frac{\delta}{2} \notin (0, 1]$. Thus, $g^{-1}(S) = (0, 1]$ is not open. Since S is open but $g^{-1}(S)$ is not, g is not continuous.

(b) In a matrix form, we hav $\begin{pmatrix} 2 & -3 & 0 \\ 4 & -6 & 1 \\ 1 & 10 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \\ 1 \end{pmatrix}$. By using Cramer's rule, we have $x = \dfrac{\begin{vmatrix} 2 & -3 & 0 \\ 7 & -6 & 1 \\ 1 & 10 & 0 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 0 \\ 4 & -6 & 1 \\ 1 & 10 & 0 \end{vmatrix}} = \frac{-23}{-23} = 1$. Similarly, we have

$y = \dfrac{\begin{vmatrix} 2 & 2 & 0 \\ 4 & 7 & 1 \\ 1 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 0 \\ 4 & -6 & 1 \\ 1 & 10 & 0 \end{vmatrix}} = \frac{0}{-23} = 0$ and $z = \dfrac{\begin{vmatrix} 2 & -3 & 2 \\ 4 & -6 & 7 \\ 1 & 10 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 0 \\ 4 & -6 & 1 \\ 1 & 10 & 0 \end{vmatrix}} = \frac{-69}{-23} = 3$.

Question 4

(1) Since $E(n^{-1}\sum_{i=1}^{n} Z_i) = E(Z_1)$, it follows from the Chebychev's inequality that for any $\varepsilon > 0$

$$P\left(\left|\frac{1}{n}\sum_{i=1}^{n} Z_i - E(Z_1)\right| \geq \varepsilon\right) \leq \frac{1}{\varepsilon^2} V\left(\frac{1}{n}\sum_{i=1}^{n} Z_i\right) = \frac{V(Z_1)}{n\varepsilon^2} \to 0, \quad n \to \infty,$$

where the last convergence to 0 is from the fact that $V(Z_1)$ is finite since $E(Z_1^2) < \infty$.

(2) Similarly as (1), we have

$$P\left(\left|\frac{1}{n}\sum_{i=1}^{n}(Z_i + \tilde{Z}_i)^2 - E\{(Z_1 + \tilde{Z}_1)^2\}\right| \geq \varepsilon\right) \leq \frac{1}{\varepsilon^2} V\left(\frac{1}{n}\sum_{i=1}^{n}(Z_1 + \tilde{Z}_1)^2\right) = \frac{V\{(Z_1 + \tilde{Z}_1)^2\}}{n\varepsilon^2}.$$

From $V\{(Z_1 + \tilde{Z}_1)^2\} \leq E\{(Z_1 + \tilde{Z}_1)^4\} \leq C \max(E(Z_1^4), E(\tilde{Z}^4)) < \infty$ where C is a constant, the above right-hand side converges to 0.

(3) Note that $E(T_i) = E\{2(2A_i - 1)\} = 4E(A_i) - 2 = 0$ and $E(T_iA_i) = 4E(A_i^2) - 2E(A_i) = 1$, which follows $E(T_iY_i \mid X_i) = E\{T_i(X_i\alpha + A_iX_i\beta + \varepsilon_i) \mid X_i\} = E(T_i)X_i\alpha + E(T_iA_i)X_i\beta + E(T_i)E(\varepsilon_i) = X_i\beta$.

(4) From the definition, $\hat{\beta}$ meets the first order KKT condition that

$$\sum_{i=1}^{n} X_i(T_iY_i - X_i\hat{\beta}) = 0. \tag{a}$$

Then $\hat{\beta} = \frac{1}{n}\sum_{i=1}^{n} X_iT_iY_i / \frac{1}{n}\sum_{i=1}^{n} X_i^2$. From the law of large numbers, we have $\frac{1}{n}\sum_{i=1}^{n} X_i^2 \xrightarrow{P} E(X_i^2) = 1$ and $\frac{1}{n}\sum_{i=1}^{n} X_iT_iY_i = \frac{1}{n}\sum_{i=1}^{n} T_iX_i(X_i\alpha + A_iX_i\beta + \varepsilon_i) \xrightarrow{P} E(T_1X_1^2)\alpha + E(T_1X_1^2A_1)\beta + E(T_1X_1\varepsilon_1)$. Since $E(T_1X_1^2) = E(T_1)E(X_1^2) = 0$, $E(T_1X_1^2A_1) = E(T_1A_1)E(X_1^2) = 1$ and $E(T_1X_1\varepsilon_1) = 0$, we obtain the result $\hat{\beta} \xrightarrow{P} \beta$.

(5) From the equation (a), $0 = \sum_{i=1}^{n} X_i(X_i\hat{\beta} - T_iY_i) = \sum_{i=1}^{n} X_i\{X_i\hat{\beta} - T_i(X_i\alpha + A_iX_i\beta + \varepsilon_i)\} = \sum_{i=1}^{n} X_i\{X_i(\hat{\beta}-\beta) + X_i\beta(1 - T_iA_i) - T_iX_i\alpha - T_i\varepsilon_i\}$. Then, $\sqrt{n}(\hat{\beta}-\beta) = \frac{1}{\frac{1}{n}\sum_{i=1}^{n} X_i^2} \cdot \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\{X_i^2(T_iA_i - 1)\beta + T_iX_i^2\alpha + T_iX_i\varepsilon_i\}$. Now, since $\frac{1}{n}\sum_{i=1}^{n} X_i^2 \xrightarrow{P} 1$, by the Slutsky's theorem and the central limit theorem, the asymptotic variance of $\sqrt{n}(\hat{\beta}-\beta)$ is $V\{X_1^2(T_1A_1 - 1)\beta + T_1X_1^2\alpha + T_1X_1\varepsilon_1\} = E\{X_1^4(T_1A_1 - 1)^2\}\beta^2 + E(T_1^2X_1^4)\alpha^2 + E(T_1^2X_1^2\varepsilon_1^2) + 2E\{X_1^4(T_1A_1 - 1)T_1\}\alpha\beta = 3(\beta + 2\alpha)^2 + 4$.

(6) From the definition, $(\tilde{\alpha}, \tilde{\beta})$ meets the first order KKT condition that

$$\begin{cases} \sum_{i=1}^{n} X_i(Y_i - X_i\tilde{\alpha}_i - A_iX_i\tilde{\beta}) = 0 \\ \sum_{i=1}^{n} A_iX_i(Y_i - X_i\tilde{\alpha}_i - A_iX_i\tilde{\beta}) = 0 \end{cases}$$

Substituting $Y_i = X_i\alpha + A_iX_i\beta + \varepsilon_i$ and simplifying leads to

$$\begin{pmatrix} \sqrt{n}(\tilde{\alpha} - \alpha) \\ \sqrt{n}(\tilde{\beta} - \beta) \end{pmatrix} = \begin{pmatrix} \frac{1}{n}\sum_{i=1}^{n} X_i^2 & \frac{1}{n}\sum_{i=1}^{n} A_iX_i^2 \\ \frac{1}{n}\sum_{i=1}^{n} A_iX_i^2 & \frac{1}{n}\sum_{i=1}^{n} A_i^2X_i^2 \end{pmatrix}^{-1} \begin{pmatrix} \frac{1}{\sqrt{n}}\sum_{i=1}^{n} X_i\varepsilon_i \\ \frac{1}{\sqrt{n}}\sum_{i=1}^{n} A_iX_i\varepsilon_i \end{pmatrix} = \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}^{-1} \begin{pmatrix} \frac{1}{\sqrt{n}}\sum_{i=1}^{n} X_i\varepsilon_i \\ \frac{1}{\sqrt{n}}\sum_{i=1}^{n} A_iX_i\varepsilon_i \end{pmatrix} + o_P(1).$$

Therefore we obtain $\sqrt{n}(\tilde{\beta}-\beta) = -\frac{2}{\sqrt{n}}\sum_{i=1}^{n} X_i\varepsilon_i + \frac{4}{\sqrt{n}}\sum_{i=1}^{n} A_iX_i\varepsilon_i + o_P(1) = \frac{1}{\sqrt{n}}\sum_{i=1}^{n} T_iX_i\varepsilon_i + o_P(1)$. Thus, the asymptotic variance of $\sqrt{n}(\tilde{\beta} - \beta)$ is $V(T_1X_1\varepsilon_1) = E(T_1^2X_1^2\varepsilon_1^2) = 4$. Compared with (5), the asymptotic variance of $\hat{\beta}$ is strictly larger than that of $\tilde{\beta}$ unless $\beta + 2\alpha = 0$.

Question 5-A.

(1) Fixed regime. Expansionary fiscal policy puts an appreciation pressure on the exchange rate. To fix the exchange rate, the central bank conducts monetary expansion. This reinforces the impact of expansionary fiscal policy on GDP. In contrast, under a floating regime, appreciation is likely to reduce net exports, which counteracts the positive impact of expansionary fiscal policy on GDP.

(2) The Balassa-Samuelson effect is an empirical observation that there is a positive correlation between income levels (or productivity of the tradable sector) and price levels across countries. In the two-country two-sector case, the assumptions are that an advanced economy has higher productivity in the tradable sector than a developing country, while there is no productivity gap in the non-tradable sector. Higher productivity in the tradable sector translates into a higher wage in the advanced economy. This pushes up the non-tradable goods price. In contrast, lower productivity of the tradable sector in the developing country implies lower wage and lower non-tradable goods price. Assuming the law of one price for tradable goods, the gap in the non-tradable goods price implies that the advanced economy has a higher price level than the developing country.

(3) Covered interest parity implies

$$R = \frac{F}{S}R^* \Rightarrow F = \frac{R}{R^*}S = \frac{1.01}{1.03}122 = 119.63 \ldots$$

(4) The dollar foreign asset is 50 trillion yen, and the dollar foreign liability is 40 trillion yen. Multiplying both by 1.2, the dollar foreign asset is 60 trillion yen, and the dollar foreign liability is 48 trillion yen. This means that the dollar-denominated net foreign asset increases from 10 trillion yen to 12 trillion yen, and the total net foreign asset increases by 2 trillion yen.

(5) Relative PPP implies the constant real exchange rate:

$$q = \frac{\varepsilon_t P_t^*}{P_t} = \frac{\varepsilon_{t-1} P_{t-1}^*}{P_{t-1}} \Rightarrow \frac{\varepsilon_t}{\varepsilon_{t-1}} = \frac{1+\pi_t}{1+\pi_t^*} = \frac{1.01}{1.05} = 0.9619 \ldots$$

This implies about 4% appreciation of the JPY and depreciation of USD.

Question 5-B

(1) Deriving Home's import demand MD_f and Foreign's export supply XS_f^*

$$MD_f = q_f^D - q_f^S = (11 - p_f) - 0 = 11 - p_f$$

$$XS_f^* = q_f^{S*} - q_f^{D*} = 6 - \left(11 - p_f\right) = p_f - 5$$

(2) Home's gains from trade under free trade

$MD_f = XS_f^*$: $11 - p_f = p_f - 5 \quad \Rightarrow \quad p_f^W = 8, \quad MD_f = XS_f^* = 3$

By symmetry the h-market has the same price and quantity: $p_h^W = 8, \quad MD_h^* = XS_h = 3$

Home's consumer surplus from importing f: $CS_f = (11 - 8) \times 3 \times \frac{1}{2} = \frac{9}{2}$

Home's producer surplus from exporting h: $PS_h = (8 - 5) \times 3 \times \frac{1}{2} = \frac{9}{2}$

Hence Home's gains from trade under free trade are $CS_f + PS_h = \frac{9}{2} + \frac{9}{2} = 9$

(3) Tariff game and payoff matrix

Home imposes a specific tariff $t \in \{0,2\}$ on imports of f; Foreign imposes $t^* \in \{0,2\}$ on imports of h. In the f-market, world price adjusts so that

$$11 - \left(p_f^W + t\right) = p_f^W - 5 \quad \Rightarrow \quad p_f^W = 8 - \frac{t}{2}, \quad MD_f = XS_f^* = 3 - \frac{t}{2}$$

$$11 - (p_h^W + t^*) = p_h^W - 5 \quad \Rightarrow \quad p_h^W = 8 - \frac{t^*}{2}, \quad MD_h^* = XS_h = 3 - \frac{t^*}{2}$$

Thus, Home's payoff is $\Pi(t, t^*) = CS_f(t) + TR(t) + PS_h(t^*)$

By symmetry, Foreign's payoff is $\Pi^*(t, t^*) = CS_h^*(t^*) + TR^*(t^*) + PS_f^*(t)$

Example: $\Pi(2,0) = CS_f(2) + TR(2) + PS_h(0) = 2 + 4 + \frac{9}{2} = \frac{21}{2}$

	$t^* = 0$	$t^* = 2$
$t = 0$	$(9\,,9)$	$\left(\frac{13}{2}, \frac{21}{2}\right)$
$t = 2$	$\left(\frac{21}{2}, \frac{13}{2}\right)$	$(8\,,8)$

(4) Nash equilibrium and discussion

Nash equilibrium: $(t, t^*) = (2,2)$ (both countries' payoffs are 8)

Under free trade $(0,0)$, both countries' payoffs are 9, whereas at the Nash equilibrium $(2,2)$both countries impose tariffs and each payoff is 8. When both impose tariffs, the volume of trade contracts and both countries are worse off than under free trade. In a situation where only one country imposes a tariff—for example $(2,0)$—the imposing country's payoff rises while the partner's payoff falls; consequently, as each pursues its own payoff, the equilibrium is $(2,2)$, at which both countries end up with smaller payoffs than if both had chosen free trade.

Question 6

This question is intended to assess the ability to discuss the relationship between labor mobility and the economy by using historical events as the subject. Candidates are expected to examine, on the basis of concrete historical facts, how population movements, changes in the supply of labor, and transformations in production structures affected the economy in a specific region or country.

Evaluation Points

1. **Presentation of a concrete case**
   Whether a specific region or country is clearly identified, and an example of labor mobility is substantively discussed.
2. **Use of historical facts as the basis of argument**
   Whether the discussion is supported by concrete facts such as dates, institutions, or events, rather than abstract generalizations, and whether those facts are accurate.
3. **Economic impact**
   Whether the effects of labor mobility on the economy are analyzed from multiple perspectives, including production, distribution, markets, and social structure.
4. **Logical organization**
   Whether the causal relationship between labor mobility and economic change is made explicit and the essay is organized in a logically coherent manner.

In addition to the above points, answers that include comparisons with other regions, a distinction between short-term and long-term effects, or an analysis of the institutional background will be evaluated more highly.

Question 7

(1) The question asks for a general historical understanding of the history of economics. The answer must summarize the development of value theory or the quantity theory of money from the eighteenth century to the first half of the twentieth century.

(a) Value Theory:
Beginning with Adam Smith and proceeding to David Ricardo, Karl Marx and others, summarize the distinctive features of each theory. Then discuss, for example, the differences between Smith and Ricardo, as well as between the classical economists and Marx. In addition, refer to the so-called "marginal revolution" that occurred in the latter half of the nineteenth century, and briefly explain the theories of Jevons, Walras, Menger, and possibly others. Conclude by referring to Alfred Marshall and the discussions of other economists in the first half of the twentieth century.

(b) Quantity Theory of Money: Summarize the historical development of the quantity theory of money, briefly referring to its main ideas. Begin with figures such as David Hume and outline the views that emerged among and after the classical economists. As for twentieth-century discussions, include Irving Fisher's equation and the debates within the Cambridge school. (Since the focus is limited to the first half of the twentieth century, it is not necessary to discuss the postwar theories of Milton Friedman and other neoclassical scholars.)

(2) Discuss the theories and/or ideas about 'poverty', 'education' or 'technological progress' advanced by one of the figures in the history of economics. Unlike the first question, this question is designed to assess understanding about measures the specific field of the history of economics (or economic thought).