２０２４年度実施

慶 應 義 塾 大 学 大 学 院 入 試 問 題

経済学研究科（修士課程）

２０２４年７月７日 実施

受験番号	Examination number	氏名	Name

注意事項（Please note:)

１. This set of questions contains 10 pages (including the cover page).

２. There are seven questions from which you should choose two to answer. Each question should be answered on a separate answer sheet. Please write the number of the question you are answering on each answer sheet.

３. If you answer two or more questions on one answer sheet, only the first answer will be treated as a valid answer. Everything after the first answer will not be marked.

４. Answer in English.

５. Although the question sheets will not be collected after the examination, please write your name and examination number (受験番号, jyuken-bango) on the cover page.

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Question 1. Answer only one of the following two questions; A and B. If you answer both, all answers for Question 1 become invalid.

A. Answer all questions in A-1 and A-2.

A-1. Consider a pure exchange economy with two goods (x_1, x_2) and two consumers (A, B). The endowment vectors are $(10, 0)$ for consumer A and $(0, 5)$ for consumer B. Let x_2 be the numeraire and the price vector be $(p, 1)$. The utility functions of consumers are:

(a) $u^A(x_1^A, x_2^A) = \ln(x_1^A) + \ln(x_2^A), \quad u^B(x_1^B, x_2^B) = \ln(x_1^B) + \ln(x_2^B)$

(b) $u^A(x_1^A, x_2^A) = \ln(x_1^A) + x_2^A, \quad u^B(x_1^B, x_2^B) = \ln(x_1^B) + x_2^B.$

Answer the following questions for both (a) and (b).

(i) Derive each consumer's demand function of the first good, $x_1^A(p)$, $x_1^B(p)$ and the competitive equilibrium price p^*.

(ii) Derive the equation for the contract curve and illustrate it using an Edgeworth box. Also, draw in the diagram ω that indicates initial endowments, E that indicates the competitive equilibrium allocation, and the budget line that realizes the competitive equilibrium allocation.

A-2. The demand function for two differential goods (q_1, q_2) is given by

$$q_1 = a - bp_1 + dp_2, \quad q_2 = a + dp_1 - bp_2.$$

Here, $a > 0,\ b > 0,\ |d| < b$. The marginal cost of both goods, $c > 0$, is constant, and the fixed cost is zero. Also, let $a > c,\ a - (b - d)c > 0$. Answer the following questions.

(a) Are two goods substitutes or complements? Answer with the sign of d.

(b) When firm 1 produces good 1 and firm 2 produces good 2, derive the Bertrand-Nash equilibrium price (p_1^*, p_2^*).

(c) When a monopolist produces two goods, derive the equilibrium price (p_1^m, p_2^m).

(d) Compare the magnitude of p_i^* and p_i^m $(i = 1, 2)$. Explain intuitively why this is so by relating it to the conclusion in (a).

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B. Consider the profit maximization problem for a firm that involves adjustment costs related to investment. The problem can be formulated as follows:

$$\max_{I_t, K_{t+1}} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \pi(I_t, K_t)$$
$$\text{s.t.} \quad K_{t+1} = (1-\delta)K_t + I_t$$

Time is discrete, denoted by t. The profit at time t is given by $\pi(I_t, K_t) = f(K_t) - I_t - \Phi(I_t/K_t)K_t$. $f(K_t)$ is the production function and we assume $\frac{\partial f}{\partial K} > 0$, $\frac{\partial^2 f}{\partial K^2} < 0$. I_t represents gross investment and K_t represents the capital stock at each time point. K_0 is given as an initial condition. The capital stock depreciates at a rate of δ, and the capital accumulation equation is $K_{t+1} = (1-\delta)K_t + I_t$. We assume that investment adjustment costs are incurred, represented by $\Phi(I_t/K_t)K_t$. The interest rate is denoted by r and remains constant over time.

1. Solve the above problem with Lagrange multiplier q_t, and describe the economic interpretation of q_t.
2. Derive the investment function assuming investment adjustment costs follow the function $\Phi(I_t/K_t) = \frac{1}{2}\left(\frac{I_t}{K_t} - \delta\right)^2$.
3. Explain the steady state for K_t and q_t, where K_t and q_t are constant, using equations and diagrams.

Question 2.

Answer the following two questions on capitalism. Base your answer on the methodology of Marxian economics.

(1) Briefly explain the following concepts.

① Two factors of commodity and dual character of the labor embodied in commodities
② Relative surplus-value
③ Relative surplus population
④ Fixed capital and circulating capital

(2) Discuss the law of average profit in the monopoly stage of capitalism.

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Question 3.

Answer the following questions on n observations $\{(Y_i, X_{i1}, X_{i2})\}_{i=1}^n$ following a linear regression model $Y_i = X_{i1}\beta_1 + X_{i2}\beta_2 + \varepsilon_i$ $(i = 1, \ldots, n)$ with two explanatory variables. Let $S_{jk} = \frac{1}{n}\sum_{i=1}^n X_{ij}X_{ik}$, $W_j = \frac{1}{n}\sum_{i=1}^n X_{ij}Y_i$ $(j = 1, 2; k = 1, 2)$ and assume $S_{11}S_{22} - S_{12}^2 > 0$.

(1) Consider estimating the coefficients $(\beta_1, \beta_2)'$ by minimizing the sum of squared residuals $L = \sum_{i=1}^n (Y_i - X_{i1}\beta_1 - X_{i2}\beta_2)^2$. Then, express the normal equation

$$\begin{cases} \frac{\partial L}{\partial \beta_1} = 0 \\ \frac{\partial L}{\partial \beta_2} = 0 \end{cases}$$

using $S_{11}, S_{12}, S_{22}, W_1, W_2, \beta_1, \beta_2$.

(2) Let $(\hat{\beta}_1, \hat{\beta}_2)'$ be the solution of the normal equation obtained by (1) and let $\hat{Y}_i = X_{i1}\hat{\beta}_1 + X_{i2}\hat{\beta}_2$. Then, show $\sum_{i=1}^n X_{i1}(Y_i - \hat{Y}_i) = \sum_{i=1}^n X_{i2}(Y_i - \hat{Y}_i) = 0$.

(3) Express $\frac{1}{n}\sum_{i=1}^n Y_i\hat{Y}_i$ using $S_{11}, S_{12}, S_{22}, \hat{\beta}_1, \hat{\beta}_2$.

(4) Suppose $(X_{i1}, X_{i2}, \varepsilon_i)'$ is an i.i.d. random vector satisfying that $E(X_{i1}) = E(X_{i2}) = E(\varepsilon_i) = 0$, each element has finite second moment and $(X_{i1}, X_{i2})'$ is independent of ε_i. Let $\Sigma_{jk} = E(S_{jk})$, $\sigma^2 = E(\varepsilon_i^2)$ and assume $\Sigma_{11}\Sigma_{22} - \Sigma_{12}^2 > 0, \sigma^2 > 0$. Then, answer the followings.

(a) Show $\frac{1}{n}\sum_{i=1}^n (Y_i - \hat{Y}_i)^2 \xrightarrow{P} \sigma^2$ if $\hat{\beta}_j \xrightarrow{P} \beta_j, n \to \infty$ for any $j = 1, 2$, where $\xrightarrow{P}$ means the convergence in probability.

(b) Let $\tilde{\beta}_1 = \frac{1}{nS_{11\cdot 2}}\sum_{i=1}^n Y_i(X_{i1} - \frac{S_{12}}{S_{22}}X_{i2})$ be an estimator for β_1 where $S_{11\cdot 2} = S_{11} - S_{12}^2/S_{22}$. Then, verify $\tilde{\beta}_1 \xrightarrow{P} \beta_1, n \to \infty$.

(c) Derive the asymptotic variance of $\sqrt{n}(\tilde{\beta}_1 - \beta_1)$ where $\tilde{\beta}_1$ is defined in (b).

Question 4.

Answer only one of A and B. If you answer two, all answers to Question 4 become invalid.

A

Answer all (1) - (3) below.

(1) Choose all of the correct statements out of the following four.

(a) Suppose that the covered interest rate parity holds and that the interest rates remain unchanged. Then, as the yen depreciates against the dollar, the yen-per-dollar forward exchange rate increases.

(b) Suppose that the relative purchasing power parity holds. Then, in a common currency area, the lower the inflation rate relative to the other countries, the more the real exchange rate depreciates.

(c) Suppose that 30% of the shares of Japanese firms are held by non-residents. In this case, when Japanese stock prices rise, the net foreign assets increase, other things being equal.

(d) When the relative purchasing power parity holds, the real exchange rate is equal to one.

(2) In the mid-1990s, the nominal yen-dollar exchange rate reached 90 yen to the dollar. Suppose that since then, the price level in Japan has remained unchanged while the price level in the U.S. has doubled. Suppose also that the relative purchasing power parity always holds. Then, what is the current nominal exchange rate (yen per dollar)?

(3) Suppose that all of Japan's foreign exchange reserves are held in U.S. Treasury bonds and financed by the issuance of Japanese short-term government bonds. Suppose also that the interest rate on Japanese short-term government bonds is 0% and the interest rate on U.S. Treasury bonds is 5%. If the exchange rate is expected to remain unchanged, what is the expected annualized return on the foreign exchange reserves? What if the uncovered interest rate parity holds?

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B

Assume that countries **X** and **Y** produce a homogeneous primary product called *Miracle Berry*, which they export globally. There are no other countries producing *Miracle Berry*. It is assumed that all *Miracle Berry* production in countries **X** and **Y** is for export only. The cost function for producing *Miracle Berry* in both countries is identical, defined as $C(q) = cq$, where q is the quantity produced, and c is the marginal cost. Additionally, the demand function for *Miracle Berry* in the importing countries is given as $D(p) = 1500 - 50p$, where p is the price.

(1) Suppose countries **X** and **Y** have formed an international production cartel to produce *Miracle Berry*. If the observed price of *Miracle Berry* is $p = 25$, estimate the marginal cost c in the cost functions of both countries. Assume that under the cartel, countries **X** and **Y** determine the total production quantity that maximizes their combined profits and allocate this total quantity equally between the two countries.

(2) Determine the producer surplus for countries **X** and **Y** under the cartel agreement, which is equivalent to the total profits obtained by these countries. Also, calculate the consumer surplus in the importing countries.

At some point, the emerging country **Z** entered the production of *Miracle Berry*, leading to the collapse of the cartel agreement between countries **X** and **Y**. Assume that the production quantity z of *Miracle Berry* by country **Z** is independently determined by the government of **Z**. Countries **X** and **Y** now face the residual demand after subtracting **Z**'s production quantity and determine their production quantities through Cournot competition.

(3) If the production quantity of *Miracle Berry* by country **Z** is $z = 100$, calculate the production quantities of *Miracle Berry* by countries **X** and **Y** respectively. Assume that the cost functions of countries **X** and **Y** do not change before and after the entry of country **Z**. Also, by what percentage does the price of *Miracle Berry* fall after the entry of country **Z** compared to before its entry?

(4) Now, consider a counterfactual scenario where countries **X** and **Y** had decided their production quantities of *Miracle Berry* through Cournot competition, even before the entry of country **Z**. Determine the price of *Miracle Berry* in this situation. Additionally, calculate the producer surplus for countries **X** and **Y** and the consumer surplus in the importing countries under this assumption, and quantitatively discuss the social welfare loss due to the cartel.

(5) Next, consider another counterfactual situation where the cartel between countries **X** and **Y** was maintained even after the entry of country **Z**. Determine the price of *Miracle Berry* in this context. Furthermore, considering the results from problem (4) above, quantitatively discuss whether the observed decline in the price of *Miracle Berry* after the entry of country **Z** is mainly due to (a) the collapse of the cartel and the occurrence of Cournot competition, or (b) the increase in the supply of *Miracle Berry* due to the entry of country **Z**.

Question 5.

Discuss the role of technological innovation in the economic development of a region or country from an economic history perspective, based on specific historical facts.

Question 6.
Choose one of the keywords below and explain the development of theory or idea in the history of economics (or the history of economic thought) which is relevant to it (Write the number of the selected keyword at the beginning of your answer).
* If you choose more than one keyword, your answer will be invalid.

Keyword:

1) Value
2) Labor
3) Profit
4) Capital
5) Money
6) Interest(rate)
7) (Natural) environment
8) Nation (or community)
9) Technological innovation
10) Knowledge (or information)

Question 7.

Consider the following definitions:

- Let $\{x_n\}_{n=1}^{\infty}$ be a sequence of real numbers. The sequence $\{x_n\}_{n=1}^{\infty}$ **converges to** x if for any $\varepsilon > 0$, there is N such that

$$n > N \Rightarrow |x_n - x| < \varepsilon.$$

- Let f be a function from the set of real numbers to itself. The function f is **continuous at** $\bar{x}$ if for any $\varepsilon > 0$, there is $\delta > 0$ such that

$$|x - \bar{x}| < \delta \Rightarrow |f(x) - f(\bar{x})| < \varepsilon.$$

(a) Using the definitions above, prove the following statements:

1. Let $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ be sequences of real numbers with limits x and y, respectively. Then the sequence $\{x_n + y_n\}_{n=1}^{\infty}$ converges to the limit $x + y$.
2. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence of real numbers with limit x^*, and f be a function from the set of real numbers to itself which is continuous at x^*. Then

$$\lim_{n\to\infty} f(x_n) = f(x^*).$$

(b) For the following matrix A, find nonsingular matrix P and diagonal matrix D so that $P^{-1}AP = D$:

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$$

(c) Find the general solution of the following system of difference equations:

$$\begin{aligned} x_{n+1} &= x_n - y_n \\ y_{n+1} &= 2x_n + 4y_n \end{aligned}$$

(d) Find the general solution of the following system of differential equations:

$$\begin{aligned} \dot{x} &= x - y \\ \dot{y} &= 2x + 4y \end{aligned}$$