

2021年度実施

慶應義塾大学大学院入試問題

経済学研究科（修士課程）

2021年7月11日 実施

科目名	Economics (English)	受験番号	Examination number	氏名	Name
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注意事項 (Please note:)

1. This set of questions contains 10 pages (including the cover page).
2. There are six questions from which you should choose two to answer. Each question should be answered on a separate answer sheet. Please write the number of the question you are answering on each answer sheet.
3. If you answer two or more questions on one answer sheet, only the first answer will be treated as a valid answer. Everything after the first answer will not be marked.
4. Answer in English.
5. Although the question sheets will not be collected after the examination, please write your name and examination number (受験番号, *juken-bango*) on the cover page.

Question 1. Answer only one of A and B. If you answer both, all answers for Question 1 become invalid.

A. Answer all questions A-1, A-2, and A-3.

A-1. (a) Firm 1 produces widgets using capital and labor. If it uses k units of capital and ℓ units of labor, the amount q_1 of widgets it can produce is expressed as

$$q_1 = (k)^{\frac{1}{2}} \cdot (\ell)^{\frac{1}{4}}.$$

Let us normalize the price of capital as 1, and let the price of labor be 4. There is no fixed cost. The firm incurs only the variable cost to pay for capital and labor. Solve the firm's cost-minimization problem to derive the total cost function $TC_1(q_1)$ when the firm produces q_1 units of widgets.

(b) Firm 2 also produces the same widgets. Its total cost function is $TC_2(q_2) = q_2$. The products of firm 1 and firm 2 are perfect substitutes. If firm 1 sells q_1 units and firm 2 sells q_2 units, the market inverse demand function (the price that sells out $q_1 + q_2$ units) is

$$P(q_1, q_2) = 9 - (q_1 + q_2).$$

Assume that firm 1 produces first. After observing the quantity q_1 , firm 2 chooses its quantity q_2 . Derive the Stackelberg equilibrium quantities (q_1^*, q_2^*) .

A-2. (a) Write down the statement of the First Fundamental Theorem of Welfare Economics as rigorously as possible. (You must mention its assumptions regarding consumers' preferences/utility functions.)

(b) Draw a diagram of a two-consumer, two-goods pure exchange economy, in which the First Fundamental Theorem of Welfare Economics holds. Using the diagram, explain the logic of the proof of the theorem.

A-3. A strategy combination $(s_1^*, s_2^*, \dots, s_n^*)$ of a strategic form game is a *strict Nash equilibrium* if

for any player $i = 1, 2, \dots, n$, s_i^* is the unique best response to

$(s_1^*, \dots, s_{i-1}^*, s_{i+1}^*, \dots, s_n^*)$.

Prove or disprove the following statement: a strict Nash equilibrium can include a mixed strategy by some player.

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B: Assume that the stock price at period t is P_t per stock, the dividend paid to the shareholders is A per stock at any period, and the net rate of return on government bonds is r . Investors sell and buy the stock and bonds in order to maximize their expected profits. Answer the following questions

- (1) Write the no-arbitrage condition between the stock and bonds.
- (2) Solve the current stock price P_t in terms of future values in a form that is as general as possible.
- (3) Write the condition under which the stock price does not include the bubble terms.
- (4) Write the economic interpretation behind the condition derived in (3).

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Question 2.

Answer the following three questions on Marx's *Capital*.

- (1) Marxian economics has not only a price dimension but also a value dimension. What is the reason why it should be so?
- (2) Chapter 13 of volume 1 in *Capital* discusses machinery and modern industry. What were the social impacts of machinery and modern industry discussed there?
- (3) Chapter 24 of volume 1 in *Capital* discusses "primitive accumulation of capital." Explain why this discussion is important for understanding capitalism. Explain the contents of Chapter 24 of volume 1.

Question 3.

Consider a case where we have n sets of observations of two variables Y, X

$$(Y_1, X_1), \dots, (Y_i, X_i), \dots, (Y_n, X_n).$$

(a) For a simple regression model using the variable X as a regressor,

$$Y_i = \alpha_0 + \alpha_1 X_i + e_i \quad \text{Eq.(1)}$$

express the least squares estimators $\hat{\alpha}_0, \hat{\alpha}_1$ by using the sample statistics of Y and X , such as the sample averages \bar{Y}, \bar{X} , sample variances s_Y^2, s_X^2 , and sample covariance s_{XY} .

(b) For the dataset we obtained, the sample mean and variance of Y were 70 and 100 respectively, those of X were 20 and 25 respectively, while the sample covariance of Y and X_1 was 35.

Calculate the least square estimates of the two coefficients.

(c) Calculate the least square estimates of the two coefficients when both the dependent variable and regressor of the dataset used in question (b) are standardized so that their means are zero and the variances are 1.

(d) Explain what the unbiasedness of an estimator is.

(e) Explain all the assumptions required for unbiasedness of the least square estimators, $\hat{\alpha}_0$, and $\hat{\alpha}_1$.

(f) Consider that instead of Eq.(1), the following model is the true data generating model,

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + e_i \quad \text{Eq.(2)}$$

Moreover, assume that X follows a standard normal distribution.

1) In this case, answer which is larger, the least squares estimate $\hat{\alpha}_1$ in Eq.(1) or $\hat{\beta}_1$ in Eq.(2), and explain the reason why.

2) Derive the marginal effect of X on Y , using the true values $\alpha_0, \alpha_1, \alpha_2$, sample mean and variance of X .

Question 4.

Answer one of A, B, and C. If you answer more than one, all answers become invalid.

A

Stocks a and b will yield state-contingent net returns outlined in the table below. Short selling is allowed for both stocks.

	Future state of the economy		
	State 1 (probability 1/3)	State 2 (probability 1/3)	State 3 (probability 1/3)
Stock a	10%	5%	0%
Stock b	15%	0%	15%

- (1) Derive the equation for the minimum-variance frontier of portfolios consisting of stocks a and b and draw a graph of the frontier.
- (2) Compute the expected return and the standard deviation of the minimum variance portfolio.
- (3) Suppose that an investor allocates 10 dollars of initial wealth between stocks a and b . Her expected utility (EU) is given by

$$EU = E[Y] - \frac{1}{6} \text{Var}[Y],$$

where Y is her terminal wealth. Derive her optimal portfolio weights. Does her optimal portfolio coincide with the minimum variance portfolio derived in part (2)? If so, why? If not, why not? Explain the economic intuitions using a diagram.

B

Consider a method of passenger transportation between two locations. The demand for this transportation mode is given by $Q = 800 - 0.01C$, where Q is the number of passengers and C is the generalized cost per passenger including both monetary and non-monetary costs of transportation usage. Let \bar{Q} be the capacity of this transportation mode. If the number of passengers is equal to or less than \bar{Q} , each passenger incurs a constant cost of \bar{C} . If the number of passengers exceeds \bar{Q} , the per-passenger cost becomes higher due to congestion. Specifically, the relationship between the generalized cost and the number of passengers is given as follows.

$$c(Q) = \begin{cases} \bar{C} & \text{if } 0 \leq Q \leq \bar{Q} \\ \bar{C} + (Q - \bar{Q})^2 & \text{if } Q > \bar{Q} \end{cases}$$

Assuming $\bar{C} = 10000$ and $\bar{Q} = 100$, answer the following questions.

- (1) Calculate the equilibrium number of passengers.
- (2) Derive the social marginal cost of this transportation mode and provide an economic interpretation.
- (3) Calculate the socially optimal number of passengers and explain why this is different from the equilibrium number of passengers.
- (4) Use a diagram to explain the welfare loss in equilibrium.
- (5) Congestion pricing by charging additional fees to passengers is one way to achieve the social optimum. Calculate the optimal congestion charge, and discuss the advantages and disadvantages of congestion pricing compared to other methods to control congestion.

C

Answer all of the following questions relating public pension systems and population trends in Japan.

- (1) Consider an economy where individuals live for two periods, where the first period is spent working and the second period is spent in retirement. Here, no one dies during the working period and everyone dies at the end of the retirement period. At time t , there are W_t workers and R_t retirees. Workers at time t (W_t) become retirees at time $t+1$ (R_{t+1}). Assume that this is a one-good economy, where the good is perishable and consumed in the period when it was produced. Each worker produces $y_W (=12)$ units and consumes c_W units per capita, whereas each retiree produces $y_R (=1)$ units and consumes c_R units per capita. The economy has a PAYGO (pay-as-you-go) pension system with defined benefits. Each retiree receives p units per capita as the pension benefit, whose level is determined so that the consumption of each retiree is equal to half of the consumption of each worker. Each worker pays q units per capita as contribution to the pension system.
- (a) When the population is increasing while satisfying the relationship $W_t = 2R_t$, compute the amount of pension benefit for retirees (p) and contribution for workers (q).
- (b) When the population is decreasing while satisfying the relationship $W_t = 0.5R_t$, compute the amount of pension benefit for retirees (p) and contribution for workers (q).
- (2) Describe the past trend and future perspective of the size and structure of the Japanese population. Using the result of (1) above, discuss the possible problems resulting from the introduction of this PAYGO pension system in Japan. Explain how the existing public pension systems in Japan (the Employees' Pension Insurance and the National Pension) handle this problem, while discussing their funding methods.

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Question 5.

Discuss the role of population in economic development, using historical facts.

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Question 6. Choose and answer one of the following questions (if you answer more than two, all answers will be invalid):

1) Outline the formation of classical economics, under the premise that it was completed by J. S. Mill.

2) Outline the formation of Keynes' economics, formulated as the IS-LM model.

3) Outline the formation of organizational economics.