Press release

Keio University

May 27, 2022

Keio University

Development of Highly Accurate Integration AI Using Deep Learning —Highest Score Ever Achieved by AI in Symbolic Integration Tests—

A research group comprising Hazumi Kubota and Yuta Tokuoka (of the same affiliation at time of study) of the Keio University Graduate School of Science and Technology, and Assistant Professor Takahiro Yamada, and Professor Akira Funahashi of the Faculty of Science and Technology have developed AIs capable of predicting the integrated function (primitive function) from the input function subject to integration (integrand), after remarking on the similarities between the mathematical processing of integrals learned in high school and those for language translation using AIs, which have seen remarkable progress in recent years. Taking inspiration from the fact that it is possible to determine the accuracy of integration from taking a differentiated primitive function produced by AI and determining whether it is consistent with the integrand, the group developed a method for creating and training diverse AIs, and deploying those AIs capable of producing the correct answer from among these. These results demonstrate that the developed AIs were capable of integration to an accuracy of 99.79%, the highest ratio ever achieved when compared to previously developed integration tools such as Mathematica and other machine learning methods. Furthermore, investigating the characteristics of the functions learned by the AI revealed that each of the constructed models had functions to which they were suited and those to which they were less suited when performing integrations. Accordingly, these models were capable of facilitating high degrees of accuracy by solving integrations in a cross-subsidized manner. Integration is an essential process for simulations in control engineering, systems biology, and other domains, and these results are anticipated to contribute to increasingly accurate simulations in these fields.

Prior to their publication in the journal *IEEE Access*, a preliminary online version of these research results was published on the journal's website on April 29.

1. Main Points of Research

- Researchers implemented an AI based on Long Short-Term Memory (LSTM)*1 and transformer models*2 to output primitive functions with integrands as input, formulating an algorithm to select the AI models whereby the functions subject to integration were consistent with the output primitive functions when differentiated.
- AI models were trained in the task of integration to discern the primitive function which would consist of an equation obtained by multiplying or dividing an elementary function by up to five.
- The results demonstrated that, after training, the AIs were capable of performing integrations to an accuracy of 99.79% when supplied with integrands in which they had not been trained, and that this accuracy was more significant than that produced when the same integrations were solved using Mathematica, a previously-developed integration tool, or methods based on

machine learning.

- By analyzing the conversion process from AI-processed integrand functions to primitive functions, researchers demonstrated that each AI had its respective strengths and weaknesses. For example, if a particular AI was poor at integrating long integrand functions, working in a cross-subsidized manner with other AIs enabled the developed AIs to achieve high accuracy.

2. Background of Research

Symbolic integration, which is learned in high school, is not limited to the field of mathematics, but rather represents an essential process for simulations to predict the dynamics of phenomena of interest in a wide range of fields, including computer science, control engineering, mechanical engineering, and systems biology. In recent years, Dr. Guillaume Lample et al. constructed an AI-based transformer model, which takes integrands as its input and the primitive function as output, demonstrating its capability in integrating functions that could not previously be solved using computers [1]. However, the method developed by Dr. Lample et al. was not capable of constructing an AI optimized for integration, as the AI was incapable of processing information contained in equations such as the order of operations.

To overcome this, the present research group expressed input and output forms as an Abstract Syntax Tree (AST)*3, allowing for the mathematical information relevant to integrations to be taken into account. They also constructed a wide range of AIs, including LSTM-based AIs which they anticipated would rigorously learn the order of operations. In addition, the group focused on the fact that, for integrations, accuracy can be determined by differentiating the primitive function outputs using AI, and comparing this with the input integrand, creating an algorithm to select the accurate results from among the broad range of AIs created.

3. Content of Research and Results

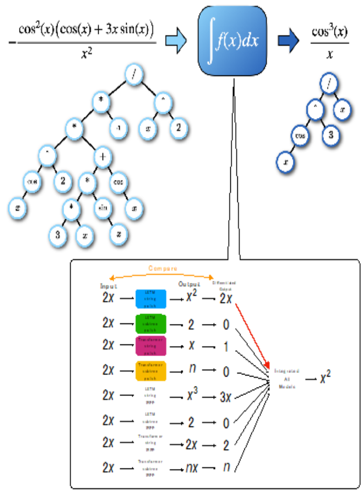

Fig. 1 Diagram of AI performing

First, researchers prepared a set of 9,697 functions as learning data. These data consist of created primitive functions obtained by multiplying or dividing up to five elementary functions (x or $\sin x$ etc.) and the paired integrand generated by differentiating created primitive functions. They developed AI which took this prepared integrand as input with the primitive function as output. Eight AIs were developed and trained in integration problems prepared for each—each AI with a different combination of the following three approaches: 1) Inputting the functions as an Abstract Syntax Tree (AST) or a word set of strings 2) Using Polish notation or reverse Polish notation read protocol for integrands 3) Using AI based on either LSTM or a transformer model. Furthermore, the team constructed an algorithm to select from these eight AIs those that were capable of producing a mathematical function consistent with the integrand after the primitive function was differentiated (Fig.1).

Using the trained AI to solve integration problems which had not been assigned during machine learning, researchers demonstrated that integration to an accuracy of 99.7% of correct responses could be achieved, an indicator of the consistency of the correct primitive function and the function of the AI output. In addition, comparisons using the same problems solved by the previously developed integration tool Mathematica and AI developed by Dr. Lample, et al, showed that the present research group's method is the most accurate.

Subsequently, the team investigated the characteristics of the integrands on which AIs focused in order to learn the reasons for the high accuracy achieved by the AIs developed in this study. As a result, they discovered that each AI had its respective strengths and weaknesses in integration such as an AI input with a simple character string which was more likely to yield errors when integrating long numerical expressions input as integrands. The method developed by this research group, whereby they selected the AI that answered correctly, has led to higher accuracies being achieved by means of cross-subsidizing the strengths and weaknesses of respective AIs.

4. Future Developments

The results of this research have shown that AI can perform highly-accurate integrations. Furthermore, detailed analyses of the functions used by the AI for integration reveal that each AI has its own respective strengths and weaknesses in solving integration problems, depending on the input method and the order it carried out operations. It can be anticipated that the application of the AI developed will lead to more accurate predictions in the future in a wider range of fields by using them in integration simulations, which in recent years have been used to predict the spread of COVID-19, among other applications.

Acknowledgments

This research was funded and supported by the Japan Science and Technology Agency (JST) JST Strategic Basic Research Program （JPMJCR2011）.

References

[1] G.Lample and F.Charton "Deep learning for symbolic mathematics," in Int. Conf. Learn. Represent. (ICLR) 2020, Addis Ababa, Ethiopia, Apr. 26-30 2020, pp. 1–24.

Details of Journal Article

Article title: Symbolic Integration by Integrating Learning Models with Different Strengths and Weaknesses

Authors: Hazumi Kubota[1]; Yuta Tokuoka[1]; Takahiro G. Yamada[1,2]; Akira Funahashi[1,2]

[1] Graduate School of Science and Technology, Keio University ; [2] Department of Biosciences and Informatics, Keio University

Journal: IEEE Access

DOI: 10.1109/ACCESS.2022.3171329

Glossary and References

*1 Long Short-Term Memory (LSTM)

Recurrent neural network capable of learning long-term dependencies of input time series data (sentences, numerical expressions, etc.)

*2 Transformer

Neural network capable of learning latent significance of inputs by automatically focusing on relationships between elements (words, operators in formulas, etc.)

*3 Abstract Syntax Tree (AST)

Data structure to express time series data with a meaningful order such as mathematical functions

*Please direct any requests or inquiries on press coverage to the contact information provided below in advance.

・Inquiries about research

Department of Biosciences and Informatics, Faculty of Science and Technology,
Keio University
Professor Akira Funahashi
TEL: +81 (0)45-566-1797
FAX: +81 (0)45-566-1789
Email: funa@bio.keio.ac.jp

・Inquiries about press release

Keio University Office of Communications and Public Relations (Contact: Sawano)
TEL: +81 (0)3-5427-1541
FAX: +81 (0)3-5441-7640
Email: m-pr@adst.keio.ac.jp
https://www.keio.ac.jp/en/